Sejny Summer Institute 2022
Pointless Causal Spaces
Causality Without Points?

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Setting the scene

The very broad general context:

Categorical Quantum Mechanics.

- Models & generalises QM using *monoidal categories*.
- Especially successful in *quantum information theory*.
- Extremely powerful and intuitive *graphical calculus*.

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What is our problem (vaguely speaking)?

Problem

No rigorous theory of *time* in CQM.

With *tensor topology*[1] we get a framework for:

Tensor topology

Space in monoidal categories.

In (very) short, we want a framework of:

Goal

*Space**time*** in monoidal categories.

[1] P. Enrique Moliner, C. Heunen and S. Tull. "Tensor Topology". In: *Journal of Pure and Applied Algebra* 224.10 (2020), p. 106378.

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Motivation: why is this interesting?

Why we want to solve this problem:

- ***Relativistic*** *categorical quantum mechanics*.
  - Quantum teleportation.
  - No-signalling.
  - No-summoning, etc.
- Fundamental nature of causality?
  - Especially in quantum theory/quantum gravity.
  - (In)definite causality?
- Applications in classical computer science (e.g. concurrency).

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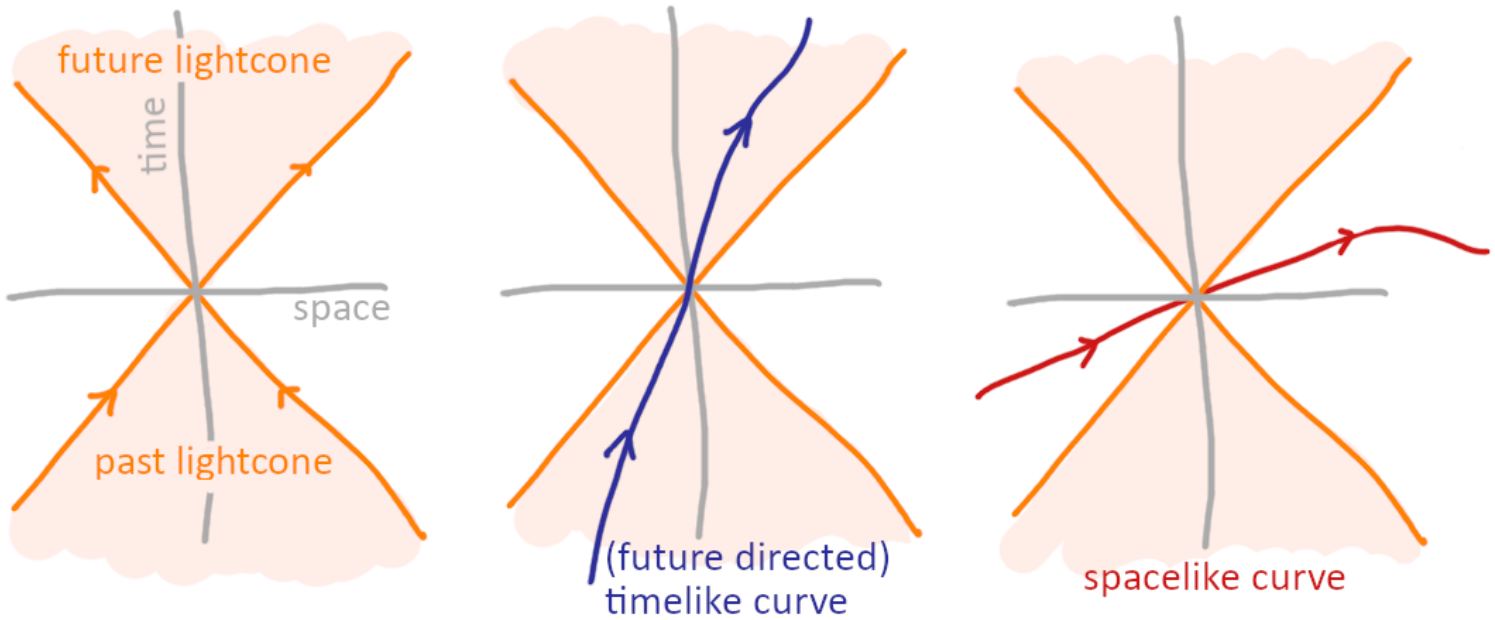
Classical causal structure of spacetime

Definition

A *spacetime* is an oriented and time-oriented connected Lorentzian manifold.

In a spacetime:



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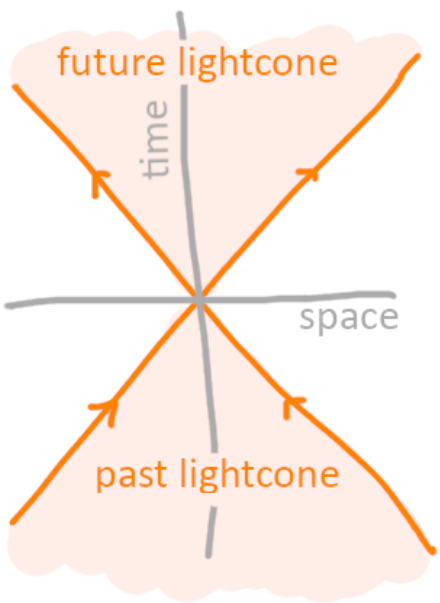
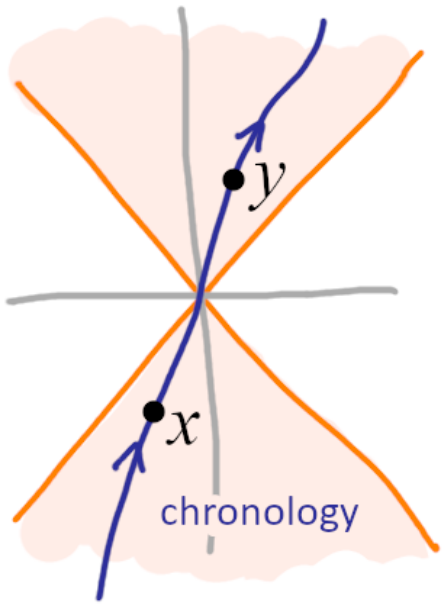
Causal structure of spacetime: causal relations

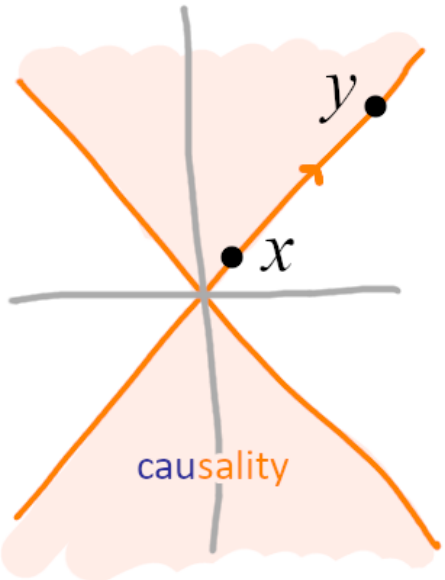
For the moment, fix a spacetime (M, g). Define:

- *Chronology:* $x \ll y$ iff there exists a smooth future directed *timelike* curve from x to y
- *Causality:* $x \prec y$ iff there exists a smooth future directed *causal* curve from x to y

We call these relations $\ll$ and $\prec$ the *causal structure* of (M, g).







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Abstract models of spacetime

So, a spacetime (M, g) induces a type of structure $(M, \prec, \ll)$.
Abstractly, such structures were first studied in

- E. H. Kronheimer and R. Penrose. "On the structure of causal spaces". In: *Mathematical Proceedings of the Cambridge Philosophical Society*. Vol. 63. 2. Cambridge University Press. 1967, pp. 481–501
- B. Carter. "Causal Structure in Space-Time". In: *General Relativity and Gravitation* 1.4 (1971), pp. 349–391

More recently (and famously):

- L. Bombelli, J. Lee, D. Meyer and R. D. Sorkin. "Space-Time as a Causal Set". In: *Physical Review Letters* 59.5 (3rd Aug. 1987), pp. 521–524

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Recovering spacetime from causal structure

It turns out that there is a strong relation between spacetime structure and causal structure.

One of the first people to study this was Zeeman:

- E. C. Zeeman. “Causality Implies the Lorentz Group”. In: *Journal of Mathematical Physics* 5.4 (1st Apr. 1964), pp. 490–493

They show the Lorentz group can be realised as *causality preserving bijections* of Minkowski space

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Recovering spacetime from causal structure

Next, in

- S. W. Hawking, A. R. King and P. J. McCarthy. “A New Topology for Curved Space-Time Which Incorporates the Causal, Differential, and Conformal Structures”. In: *Journal of Mathematical Physics* 17.2 (2 Feb. 1976), pp. 174–181
- D. B. Malament. “The Class of Continuous Timelike Curves Determines the Topology of Spacetime”. In: *Journal of Mathematical Physics* 18.7 (1st July 1977), pp. 1399–1404

it was shown that the timelike *continuous* curves in a spacetime determine its:

- Smooth structure
- Conformal structure

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Recovering spacetime from causal structure

This was improved in the paper:

- K. Martin and P. Panangaden. “A Domain of Spacetime Intervals in General Relativity”. In: *Communications in Mathematical Physics* 267.3 (1st Nov. 2006), pp. 563–586

where it was shown that the causal *relation* $\prec$ of a globally hyperbolic spacetime fully determines its topology.

This suggests that causal structure is more fundamental than its

- Smooth structure
- Conformal structure
- Topological structure

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Recovering spacetime from its causal structure

Conclusion:

Causal structure determines almost all of spacetime geometry!

So, we think, this justifies studying abstract spacetimes of the form

$$(M, \prec, \ll)$$

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Motivation: our main problem

- Causality (even abstractly) of spacetimes is actually well-studied, so:
- **Main idea:** *'categorify'* the causal structure of classical spacetime.
- **Problem:** tensor topology is *pointless*.

$$\mathrm{ZI}(\mathrm{Sh}(X)) \cong \mathcal{O}X \qquad \text{and} \qquad \mathrm{ZI}(\mathbf{Hilb}_{C_0(X)}) \cong \mathcal{O}X.$$

- Hence our typical notions of causality from spacetime do not carry over directly!
- We were led to think about causality *without* points...

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Causality without points?

This was a technical motivation.
But there is also a philosophical argument from quantum theory:

- P. Forrest. "From Ontology to Topology in the Theory of Regions*". In: *The Monist* 79.1 (1st Jan. 1996), pp. 34–50

Quantum theory is *probabilistic*:

- The probability of e.g. an electron to occupy an exact point in spacetime is *zero*
- Rather, we assign probabilities of the electron to occupy *regions*

Hence it seems quantum theory also suggests the study of causality between *regions/systems/reference frames!*

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Philosophical questions

For this summer school I want to propose the following (rather open-ended) questions:

- ***"What is causality without points, and what does that mean in (quantum) physics?"***

More specifically:

- ***"Can we relate these ideas about regions to existing notions of (indefinite) causality in QM?"***
- ***"Does it even make sense to study causality of regions in QM, or do we need something else?"***

In my mini-course I will outline the mathematics needed to study causality of regions, and highlight a new result justifying the study of *'causal locales'*.

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A more concrete proposal

Out of Sejny 2021 came a nice paper:

- P. Martin-Dussaud, T. Carette, J. Głowacki, V. Zatloukal and F. Zalamea. *Fact-Nets: Towards a Mathematical Framework for Relational Quantum Mechanics*. 1st Apr. 2022. arXiv: 2204.00335 [quant-ph]

Summary:

- A model of *relational* QM using *fact nets:*
  - Systems
  - Facts between systems

Hence I would like to investigate a more concrete problem:

- ***"Can we study (mathematically) causality in this framework?"***
- Where:
  - Systems/reference frames represent abstract regions
  - How does causality interact with the facts?

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Thank you!