Axioms for Hilbert spaces & Linear Contractions

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QS LAB WORKSHOP.
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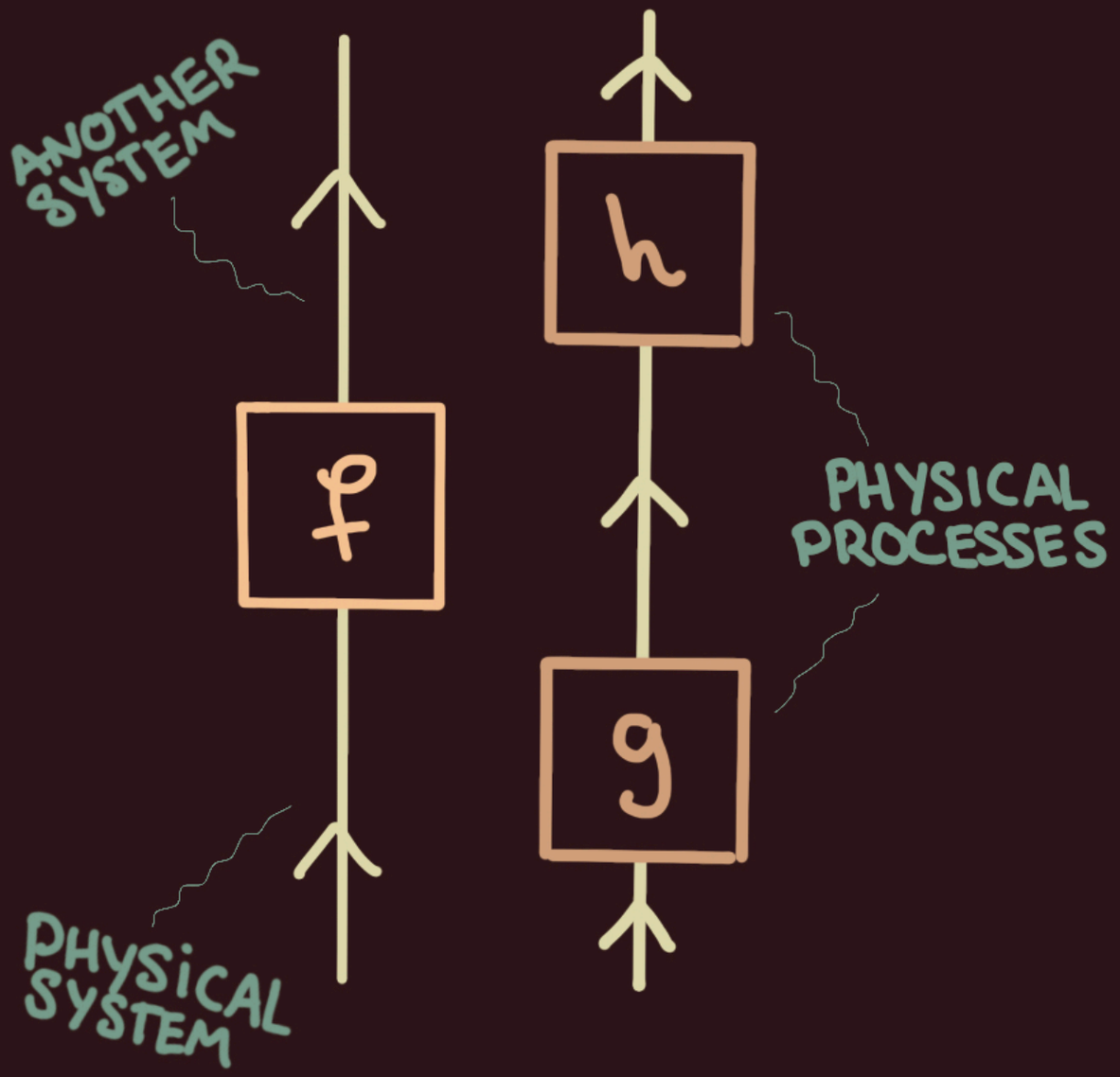
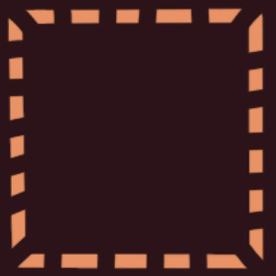
PHYSICS AS PROCESSES

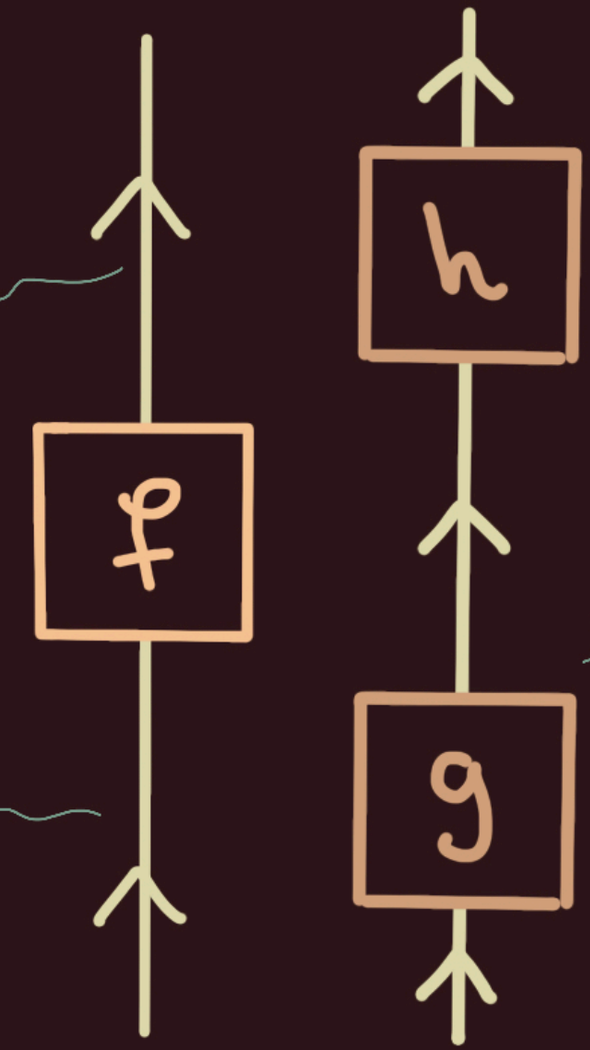


"EMPTY" SYSTEM $I =$ 

HOW CAN WE TELL OUR PROCESSES ARE

QUANTUM?

QUANTUM PROCESSES



HILBERT SPACES

CONTINUOUS/BOUNDED LINEAR OPERATORS

HILB

Quantum Processes

Mixed:	CPTP maps of operators (preserve mixed states, i.e. density matrices)
Pure:	Linear contractions of Hilbert spaces (preserve pure states)

RECOVERING HILBERT SPACES

HEUNEN + KORNELL, PNAS 2022 Vol. 119 No. 9
arXiv:2109.07418

AXIOMS:

(1) $\mathbb{C}$ IS DAGGER $(\dagger\text{-})$ MONOIDAL

ADJOINTS EXIST

$\otimes$ EXISTS

(2) I IS A SIMPLE SEPARATOR

$A(v \otimes w) = B(v \otimes w) \Longrightarrow A = B$

$\mathbb{C}$ HAS NO NON-TRIVIAL SUBSPACES

(3) $\mathbb{C}$ HAS $\dagger$- BIPRODUCTS

$\oplus$ EXISTS

RECOVERING HILBERT SPACES

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AXIOMS:

(4) $\underline{\subset}$ HAS †-EQUALISERS

(5) †-MONOS ARE †-KERNELS — CLOSED SUBSPACES ARE KERNELS

(6) $\underline{\subset}$ HAS DIRECTED COLIMITS OF †-MONOS — COMPATIBLE SUBSYSTEMS CAN BE VIEWED AS A SINGLE HILBERT SPACE

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THEOREM. IF $\subseteq$ SATISFIES AXIOMS (1)-(6) THEN

$$\subseteq \simeq \mathrm{HILB}.$$

HILBERT SPACES & CONTRACTIONS

HEUNEN + KORNELL + vdS, arXiv:2211.02688

THE IDEA:

ABSTRACT SCALARS: $f : I \longrightarrow I$

IN HILB: $\mathbb{C} \longrightarrow \mathbb{C}$

IF YOU FORMALLY INVERT SCALARS IN CON,

YOU GET HILB!

HILBERT SPACES & CONTRACTIONS

HEUNEN + KORNELL + vdS, arXiv:2211.02688

THE IDEA:

AXIOMS ON $\mathbb{D}$ SUCH THAT:

$$\mathbb{D} \hookrightarrow \mathbb{D}[\text{SCALARS}^{-1}]$$

$$\| \wr$$

$$\underline{\text{CON}} \hookrightarrow \underline{\text{HILB}}$$

HILBERT SPACES & CONTRACTIONS

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THE IDEA:

AXIOMS ON $\mathbb{D}$ SUCH THAT:

$$\begin{array}{ccc} \mathbb{D} & \hookrightarrow & \mathbb{D}[\text{SCALARS}^{-1}] \\ \wr & & \wr \\ \text{CON} & \hookrightarrow & \text{HILB} \end{array}$$

THANKS FOR YOUR ATTENTION!